

THE COUPLING OF ELECTROMAGNETIC POWER TO PLASMAS

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ABSTRACT

This lecture complements the three previous lectures^{1,2,3} on waves by addressing, on the basis of elementary and intuitive treatment, the process of coupling of electromagnetic power to plasma. Coupling is here meant in a broad sense. It consists of four different steps. (i) The first one is the coupling of vacuum electromagnetic power to plasma waves. An elementary antenna coupling theory is given. The state of the art in coupling models and status of comparisons with experiments are briefly discussed. (ii) The second is the transfer of plasma wave energy to particle energy. The resonant processes leading to this transfer are described in a heuristic way. (iii) The third one is the build-up of fast particle populations. It will be outlined through a sketch of quasilinear diffusion for the simple case of Landau damping. (iv) The last step is the conversion of power through the resonant particle population to bulk plasma heating by collisions, which will be briefly addressed.

I. INTRODUCTION

Dielectric tensor notations are taken over from the lecture by Westerhof², but like in the previous lecture⁴ we stick to the SI system, except for temperatures which will sometimes be expressed in eV, where explicitly stated in the text.

The principle of wave heating is similar for all schemes and is sketched in Fig.1. The electromagnetic energy is produced by a generator and sent to the machine area via transmission lines constituted of coaxial lines at low frequency and waveguides at higher frequency. At very high frequency optical transmission is also possible. Some matching circuitry has to be incorporated in the transmission system in order to prevent the reflected power to come back to the generator. The transmission line is connected to some launching structure (antenna, waveguide,...) that will couple the power inside the machine's vacuum chamber. The vacuum wave that exists

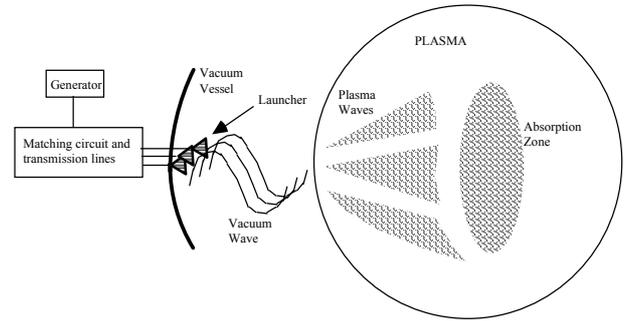


Fig. 1. Principle of heating by plasma waves

inside the launching structure and at the very edge of the plasma is then converted to a plasma wave that transports electromagnetic energy to some region inside the plasma where it will be absorbed. This is the region where the resonant process occurs. This process accelerates the population of particles that is in resonance with the wave, usually a small fraction of the plasma particles. A slightly or strongly non-maxwellian resonant population builds up against the restoring force of collisions between this population and the remainder of the plasma. It is through the latter collisional process that the bulk of the plasma is heated up.

To fix the ideas, it is useful to have in mind the typical order of magnitude of the plasma parameters. Let us consider typical parameters of a JET-type plasma: The basic frequencies and velocities are given in Table 1. Note that electron frequencies (f_{ce} , f_{pe} , f_{UH}) are much higher than all other frequencies due to the smallness of m_e/m_i .

In this introduction, we shall not describe the

TABLE 1. Plasma parameters (Deuterium)

$T=5\text{keV}$	$f_{ce}=80\text{GHz}$	$f_{LH}=0.8\text{GHz}$	$V_A=7.10^6\text{m/s}$
$n=5 \cdot 10^{19}\text{m}^{-3}$	$f_{pe}=60\text{GHz}$	$f_{ci}=23\text{MHz}$	$V_{te}=3.10^7\text{m/s}$
$B_0=3\text{T}$	$f_{pi}=1\text{GHz}$	$k_{\perp}V_A=10\text{MHz}$	$V_{ti}=5.10^5\text{m/s}$

technical parts of the launching systems, i.e. generators, transmission lines, matching systems, but shall focus on the physics of power coupling. Let us first start with the launcher theory.

II. ELEMENTARY WAVE COUPLING THEORY

II.A. Types of launcher and coupling

The simplest case is that of electron cyclotron waves in large machines. In this case, the wavelength of the vacuum wave ($k_0 = \omega/c$ is the vacuum wavevector)

$$\lambda = 2\pi / k_0 \quad (1)$$

is very small as compared to the plasma cross-section. The wave is launched as a propagating wave pencil that will progressively convert to a plasma wave. Because of the smallness of the wavelength, the boundary conditions at the conducting wall of the machine, as well as on the launching structure, play no explicit role. The wave can be accurately described in the geometric optics limit and the only boundary conditions that matter are the initial launching angle and reflections at the wall, if any.

If the vacuum wavelength becomes comparable to the antenna structure, the scale length of variations of edge plasma parameters or the plasma radius, the launcher environment and the plasma will affect the coupling process and a full boundary-value problem has to be solved to describe it. Such is usually the case of Alfvén wave, ion cyclotron, or lower hybrid wave launchers in medium or large-size machines. But this may also be the case of electron cyclotron launchers in low-field, small machines.

One can still distinguish two different cases in the latter category. Either the vacuum wave propagates inside the torus chamber, or it is evanescent. This depends first on the size of the machine relative to the wavelength. Indeed in a torus, the poloidal and toroidal wavenumbers will be quantified. To a first approximation, if we call R_0 the major radius and r_0 the minor radius of the machine, the fundamental toroidal wavenumber of the toroidal cavity will be $1/R_0$ and the poloidal one $1/r_0$. Let us call m and n the poloidal and toroidal wavenumbers respectively, then the dispersion relation for vacuum waves can be written (with k_r the radial component of the wavevector) :

$$k_r^2 = k_0^2 - m^2 / R_0^2 - n^2 / r_0^2 \quad (2)$$

The wave can propagate towards the plasma inside only if k_r is real because the fields behave as

$$\mathbf{E}, \mathbf{B} \propto \exp(\pm i k_r r) \quad (3)$$

The existence of the $m=n=0$ mode is prohibited by the metallic boundary conditions. Hence, a necessary

condition for the vacuum wave to propagate inside the torus is

$$k_0 > m / R_0 \quad (4)$$

If $k_0 < r_0$, then only the $m=0$ mode can propagate and the rest of the poloidal spectrum is evanescent. It is usually the case for medium size or large machines that part of the antenna spectrum is propagating, part evanescent. One can globally estimate in which category a launching structure will fall only from its dimensions and phasing between elements. Let us call k_y the poloidal wavenumber and $k_{||}$ the toroidal one [we shall associate to (x,y,z) the radial, poloidal and toroidal directions respectively]. If the launching structure has width w the support of the launcher field or current is (up to a translation) proportional to the function

$$S_{||}(z) = \gamma(z) - \gamma(z - w) \quad (5)$$

which power spectrum is proportional to

$$S_{||}^2(k_{||}) \propto \sin^2(k_{||} w / 2) \quad (6)$$

Therefore, the typical parallel wavenumber of the structure can be taken as $k_{||a} = \pi/w$. Similarly, if the structure has height h , the typical poloidal wavenumber is $k_{ya} = \pi/h$. Therefore, one can estimate the typical radial wavevector for the launcher from a formula similar to Eq.(2):

$$k_r^2 = k_0^2 - k_{ya}^2 - k_{||a}^2 \quad (7)$$

Assume, as is usually the case in the lower frequency range ($f \lesssim f_{ci}$) that the antenna width is much smaller than the wavelength. Then the poloidal average ($k_y=0$) of the field will decay as

$$\exp(i k_r x) = \exp(-k_{||a} x) = \exp(-\pi x / w) \quad (8)$$

This provides a simple rule to estimate the evanescence of the field launched by the antenna in vacuum. If the launcher is made of an array of identical elements spaced by $L_z < w$ in the toroidal direction and excited with a phase difference $\Delta\phi$, then the typical parallel wavevector must be taken as $k_{||a} = \Delta\phi / L_z$. (if $> \pi/w$, and similarly for the y direction). Evanescence will then be stronger than in the previous case of an unphased structure.

We shall now introduce coupling theory, on the basis of the simplest model. More sophisticated theories rest on similar principles but include more of a realistic geometry.

II.B. Coupling model

We consider the simplest case of an antenna facing a large plasma such that the plasma looks nearly uniform in the toroidal (z) and poloidal (y) directions (Fig.2). In the radial direction x the plasma is usually non-uniform, and this is taken into account in standard coupling models, but here, in order to simplify the algebra, we consider a step model. The density is zero for $x < 0$ and constant density

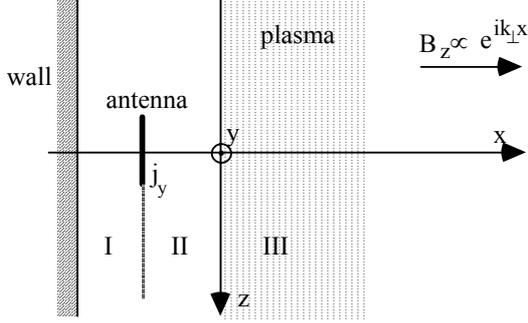


Fig. 3 2-D slab coupling model. The plasma is uniform in the y (poloidal) and z (toroidal) directions. The width of the antenna is $2w_z$, the distance between the antenna and the plasma is a and the distance between the antenna and the wall is d .

for x positive. We assume that the absorption is good and hence, there is no reflected wave. This is the so-called *single-pass* approximation. In addition, we shall also assume that the system is invariant in the y -direction (which implies in particular that the antenna is infinite) and neglect propagation in the y -direction ($k_y=0$). Next, we have to choose a model for the plasma waves. To be specific, we choose the case of coupling in the ion cyclotron frequency range (ICRF). This case was treated by Westerhof², section III.A. The slow wave being evanescent in the plasma bulk, we shall consider only coupling to the fast magnetosonic wave (FW). As the FW equations will be needed later on, we first derive them, starting from the cold wave theory².

II.C. The fast magnetosonic wave equation

We write the full cold dispersion relation² in terms of the parallel ($N_{\parallel}=N\sin\theta$) and perpendicular ($N_{\perp}=N\cos\theta$) components of the refractive index

$$N_{\parallel} = \frac{k_{\parallel}}{k_0}; \quad N_{\perp} = \frac{-i}{k_0} \frac{d}{dx} \quad (9)$$

We keep to N_{\perp} its operator meaning because x is the direction of inhomogeneity. In the parallel direction, the plasma is homogeneous and we use the Fourier transformed form. The dispersion equation can then be rewritten

$$\begin{pmatrix} S - N_{\parallel}^2 & -iD & N_{\perp} N_{\parallel} \\ iD & S - N_{\parallel}^2 - N_{\perp}^2 & 0 \\ N_{\perp} N_{\parallel} & 0 & P - N_{\perp}^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0 \quad (10)$$

It has been shown² that the uncoupling of the FW equations follows from the very large value of P . Mathematically, this amounts to taking the limit

$$P \rightarrow \infty \quad (11)$$

This is equivalent to assuming that the plasma conductivity is infinite in the parallel direction. It is also the same as taking the zero electron mass limit. This limit implies that the parallel electric field cannot penetrate the plasma, i.e. that the evanescence length of the slow wave is zero. Taking the limit Eq.(11) in Eq.(10), we obtain

$$\begin{pmatrix} k_0^2 S - k_{\parallel}^2 & -ik_0^2 D \\ ik_0^2 D & k_0^2 S - k_{\parallel}^2 + \frac{d^2}{dx^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = 0 \quad (12)$$

and $E_z=0$. The other components of the fast wave field follow from Maxwell's equation $i\omega\mathbf{B} = \nabla \times \mathbf{E}$:

$$H_x = H_y = 0; \quad H_z = \frac{1}{i\omega\mu_0} \frac{dE_y}{dx} \quad (13)$$

Finally, eliminating E_x from Eq.(12), we obtain the Fast wave equation

$$\frac{d^2 E_y}{dx^2} + k_{\perp FW}^2 E_y = 0 \quad (14.1)$$

$$k_{\perp FW}^2 = k_0^2 N_{\perp FW}^2 = (k_0^2 S - k_{\parallel}^2) - \frac{(k_0^2 D)^2}{(k_0^2 S - k_{\parallel}^2)} \quad (14.2)$$

II.D. The plasma surface impedance

For a uniform plasma, the wave equation is Eq.(14.1) has constant coefficient and the solutions are simply exponential. The single-pass approximation allows us to impose at $z \rightarrow \infty$ a *radiating boundary condition* and the wave solution in the plasma can be written:

$$E_y = C_{III} \exp(ik_{\perp} x) \quad (15)$$

where C_{III} is a constant (relative to region III in Fig.2) to be determined. Eq.(13) then gives the H_z field component:

$$H_z = \frac{ik_{\perp}}{i\omega\mu_0} E_y \quad (16)$$

The field in the plasma is thus known up to a multiplicative constant. Tangential field components being continuous at the plasma-vacuum interface (II-III), their ratio is also continuous. This quantity is known as the *surface impedance* of the plasma Z_S

$$Z_S = \frac{E_y}{H_z} = \frac{\omega\mu_0}{k_{\perp}} \quad (17)$$

We express the continuity of this quantity at $x=0$ as

$$[[Z_S]]_0 \equiv Z_S(0_+) - Z_S(0_-) = 0 \quad (18)$$

In the general case where all field components are to be considered, the equivalent of Eq.(17) is a vector relation and \mathbf{Z}_S is the surface impedance matrix:

$$\begin{pmatrix} E_y \\ E_z \end{pmatrix} = \mathbf{Z}_S \begin{pmatrix} H_y \\ H_z \end{pmatrix} \quad (19)$$

II.E. Fields in the vacuum region I-II

Equipped with this boundary condition, the vacuum problem can be solved on its own. The plasma properties will enter its solution only via the quantity Z_S and the vacuum solution is therefore formally independent of the particular plasma model considered. The general electromagnetic field in vacuum can be decomposed into its TE (transverse electric) and TM (transverse magnetic) parts with respect to a given direction, here z . Maxwell's equations then appear in the form:

$$\begin{pmatrix} B_x \\ E_y \end{pmatrix} = \frac{1}{k_0^2 - k_{||}^2} \begin{pmatrix} -i\omega & ik_{||} \\ ik_{||} & -i\omega \end{pmatrix} \begin{pmatrix} ik_y E_z \\ \frac{dB_z}{dx} \end{pmatrix} \quad (20.1)$$

$$\begin{pmatrix} E_x \\ B_y \end{pmatrix} = \frac{1}{k_0^2 - k_{||}^2} \begin{pmatrix} -i\omega & ik_{||} \\ -ik_{||} & i\omega \end{pmatrix} \begin{pmatrix} -ik_y B_z \\ \frac{dE_z}{dx} \end{pmatrix} \quad (20.2)$$

$$\frac{d^2}{dx^2} \begin{pmatrix} E_z \\ B_z \end{pmatrix} = (k_y^2 + k_{||}^2 - k_0^2) \begin{pmatrix} E_z \\ B_z \end{pmatrix} \quad (21)$$

From this it can be seen that the problem can be solved independently for E_z and B_z . The TM part of the field, which has a longitudinal (along z) \mathbf{E} component does not couple to the plasma waves because $E_z=0$ in the plasma (§II.C). Therefore, for the simplified problem considered here, we can retain the TE mode alone and ignore the field components deriving from E_z . The solution for the two vacuum regions is elementary:

$$\text{Region I: } B_z = A_I \cosh(k_{||}x) + B_I \sinh(k_{||}x) \quad (22.1)$$

$$\text{Region I: } B_z = A_{II} \cosh(k_{||}x) + B_{II} \sinh(k_{||}x) \quad (22.2)$$

where A 's and B 's are constants. to be determined by applying the following boundary conditions.

- At the metallic wall $x=-(a+d)$: $E_y = 0$ (23.1)

- The antenna is represented by an infinitely thin current sheet of finite width w and infinite length. This gives rise to a jump condition on the tangential magnetic field: $[[B_z]]_{-a} = -\mu_0 j_y$ (23.2)

- and a continuity condition $[[E_y]]_{-a} = 0$ (23.3)

- At the plasma surface, $x=0$: $[[Z_S]]_0 = 0$ (23.4)

These conditions are sufficient to determine the 4 constants in Eq.(22). In particular, this gives the relation between all field components and their source, the current density at the antenna j_y . In the general case one has a vector relation

$$\mathbf{E} = \overline{\mathbf{R}}(k_y, k_{||}) \mathbf{j} \quad (24)$$

There is one additional condition, the continuity of E_y (or B_z) at $x=0$ that was not necessary to solve the vacuum problem. It can be used to determine C_{III} in Eq.(15) as all field quantities in the vacuum region are now known.

II.F. Poynting's theorem and antenna radiation

Let us consider in the vacuum region I-II an arbitrary volume containing the antenna. Starting from Maxwell's equations, one can easily write down Poynting's theorem

$$-\frac{1}{2} \int_V \mathbf{E} \cdot \mathbf{j}_A dV = \int_S \frac{1}{2} (\mathbf{E} \times \mathbf{H}^*) dS - i\omega \int_V \left(\frac{\mu_0}{2} |\mathbf{H}|^2 - \frac{\epsilon_0}{2} |\mathbf{E}|^2 \right) dV \quad (25)$$

On the LHS appears the work done by the electric field on the antenna current. Strictly speaking, it should be zero because the antenna is a metallic conductor on which the tangential electric field should vanish. It is non-zero because the current distribution on the antenna has been *assumed* rather than self-consistently computed. This is known as the *induced e.m.f. method*. Though it may appear rough, this method usually gives good results if the assumed current is a reasonable guess of the exact one. In more sophisticated computations⁵, the current distribution on the antenna is self-consistently determined.

A theory completely similar to the above one can be done for waveguide⁶ or aperture launchers. In these cases, the incoming wave field distribution on the aperture is given and the field reflected by the plasma and surrounding structures is the result of the computation. Alternatively, the above formalism can be applied without changes if the aperture boundary condition is expressed as an equivalent current density⁷.

It is to be observed that the quantity on the LHS of Eq.(25) has both a real and an imaginary part, as explicitly apparent in the RHS. The real part is the power radiated by the antenna, while the imaginary part is related to the reactive properties of the antenna, as we will see.

II.G. Antenna coupling properties

The structure of the antenna modelled in the present exercise is basically that of a *strip-line*, i.e. a conductor running above an infinite conducting plane. The field in such a strip line is known to have a TEM (transverse electro-magnetic) structure, like a coaxial transmission line. A TEM field has the property that the electric and magnetic field structure in the transmission line cross-section is the same as that respectively of the electrostatic and magnetostatic field it can sustain. Therefore, the antenna properties can be computed in the electrostatic and magnetostatic limits and used as in transmission line theory. We shall recast the results obtained above in terms of strip line characteristics. This formalism is often used in practice to represent properties of real ICRF antennas, which structure is indeed close to that of strip lines. First, we rewrite Poynting's theorem in terms of the $k_{||}$ field spectrum using Parseval's relation:

$$P = -\frac{1}{2} \int_V \mathbf{E} \cdot \mathbf{j}_A^* dV = -\frac{1}{4\pi} \int_{-\infty}^{\infty} E_{yA}(k_{//}) j_A(k_{//}) dk_{//} \quad (26)$$

Then, we equate the computed power to the the expression of the same quantity for a strip line:

$$P = \frac{1}{2} (R - i\omega L) I^2 = -\frac{1}{4\pi} \int_{-\infty}^{\infty} E_{yA}(k_{//}) j_A(k_{//}) dk_{//} \quad (27)$$

This relation assimilates the antenna radiated power to the equivalent power of a lossy transmission line element with R and L the specific (i.e. per unit length) resistance and inductance of the line. The second equality constitutes the definition of these two quantities in the present antenna model. I is the total current flowing on the antenna. No equivalent capacitance C appears in Eq.(27) because we have dropped the TM part of the field. It can however easily be obtained by solving the TM vacuum field equations with the boundary condition $E_y=0$ at the plasma $x=0$. The three constants R, L, C completely determine the properties of the transmission-line equivalent to the antenna. In particular,

$$\beta = \omega \sqrt{LC} \quad (28.1)$$

is the propagation constant along the line. If the plasma is replaced by a metallic wall, this is simply the vacuum wave vector ω/c , as in a coaxial line. In the presence of plasma, because the magnetic field leaks out of the vacuum region into the plasma, L (proportional to the magnetic energy) is larger than the vacuum value and the wavelength of the field along the antenna is shorter. One therefore says that the antenna is a *slow-wave structure* because the propagation velocity along it ω/β is lower than c . The slow-wave structure property is due to the anisotropy of the boundary $x=0$ which behaves as a perfect conductor along z and as a dielectric in the y direction.

$$Z_c = \sqrt{L/C} \quad (28.2)$$

is the characteristic impedance of the antenna and

$$Z_A = \sqrt{\frac{R - i\omega L}{-i\omega C}} \tanh[-i\omega C(R - i\omega L)l_A] \quad (28.3)$$

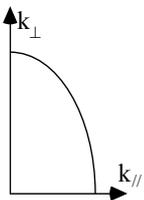


Fig. 3 Fast wave dispersion

is the antenna input impedance with l_A the antenna length. This quantity is of primary interest for the design of the transmission and matching system.

II.H. Radiated field

Using the additional boundary condition at the plasma-vacuum interface, i.e. continuity of E_y , the field in the plasma can also be computed and used to determine the properties of the radiated far field. We start from the expression of E_y solution of the simplified boundary-value problem above:

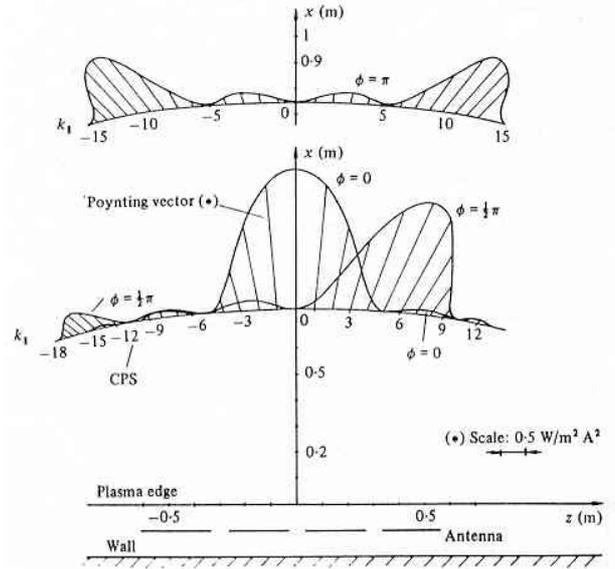


Fig. 4. Distribution of the Poynting vector in the far-field region for three different phase differences ϕ between successive straps of a 4-strap antenna array. From⁸.

$$E_y(x, k_{//}) = \frac{-Z_S \sinh(k_{//}d) j_A(k_{//}) \exp(ik_{\perp}x)}{\sinh[k_{//}(a+d)] + \frac{ik_{//}Z_S}{\omega\mu_0} \cosh[k_{//}(a+d)]} \quad (29)$$

For a simple uniform current distribution on the antenna, the current spectrum is:

$$j_A(k_{//}) = \frac{\sinh(k_{//}w/2)}{k_{//}w/2} \quad (30)$$

The real space distribution of the fields is obtained by inverse Fourier transform:

$$E_y(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_y(x, k_{//}) e^{ik_{//}z} dk_{//} \quad (31)$$

$$= \int_{-\infty}^{\infty} G(k_{//}) j_A(k_{//}) e^{i(k_{\perp}x + k_{//}z)} dk_{//}$$

where the integrand has been recast in a form suitable for using the stationary phase approximation. In the far-field region x and z are large and the exponential is varying fast while G and j_A are slowly varying functions of $k_{//}$. The main contribution to the integral thus comes from the region where the phase is stationary:

$$\frac{dk_{\perp}}{dk_{//}} x + z = 0 \quad (32)$$

The relation between k_{\perp} and $k_{//}$ is determined by the dispersion relation [see Eq.(14)]

$$k_{\perp}^2 = k_{\perp FW}^2(k_{//}) \quad (33)$$

and is shown schematically in Fig.4. Equation (32) is the equation of the rays launched from the antenna. Each ray is a straight line with angular coefficient x/z uniquely determined by the value of $k_{//}$. For $k_{//}=0$, the ray is launched perpendicularly to the wall, for nonzero $k_{//}$ it is launched at an angle and for $k_{//}$ corresponding to the cut-

off of the fast wave ($k_{\perp}=0$), it is launched in the parallel direction. The ray direction can be shown to be identical to the Poynting vector direction as

$$\mathbf{P}_x = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot \mathbf{e}_x = \frac{k_{\perp} |E_y|^2}{2\omega\mu_0} \begin{pmatrix} 1 \\ 0 \\ -\frac{dk_{\parallel}}{dk_{\perp}} \end{pmatrix} \quad (34)$$

The electric field amplitude follows from the stationary phase approximation of the integral Eq.(31):

$$E_y(x, z) = -\sqrt{\frac{2\pi}{-ix d^2 k_{\perp} / dk_{\parallel}^2}} G(k_{\parallel}) J_A(k_{\perp}) e^{i(k_{\perp} x + k_{\parallel} z)} \quad (35)$$

where all quantities are evaluated using Eq.(32). In Fig.4, the far field Poynting flux distribution over constant phase surfaces is shown for a phased antenna array in an ITER-like plasma. As the Poynting flux is the RF power flux, this shows how phasing, by sending the power at different parallel wave numbers sends it in different spatial directions as well.

II.I State of the art

Because the vacuum problem can be solved for itself, independently of the complexity of plasma properties, sophisticated antenna models have been developed⁹. They focus either on a precise determination of the antenna currents⁵, on the recesses in which antennas are located¹⁰, or on more complete description of plasma waves¹¹. Antenna models have been compared with success to experimental results in a number of cases, both in the ICRF^{10,12,13} and Lower Hybrid range¹⁴ and are used to predict the performance of ITER antennas¹⁵.

III. POWER COUPLING FROM WAVES TO PARTICLES

III.A. Absorption mechanisms

Once the wave has been launched in the plasma, one could think that it can be damped simply because the accelerated particles experience a drag due to collisions. This is in general not the case. In the bulk of a hot plasma, e.g. $T_e \approx T_i \approx 5\text{keV}$, $n = 5 \times 10^{19} \text{m}^{-3}$ the collision frequency is

$$\nu = 2.9 \times 10^{-12} n \ln \Lambda T^{-3/2} \approx 20 \text{kHz} \quad (36)$$

In electromagnetic theory the ratio ν/ω , ($\omega=2\pi f$) is characteristic of the importance of dissipative effects due to collisions with respect to reactive, i.e. wave oscillation, energy. For small values of ν/ω , the motion is almost dissipation-less and huge fields and large perturbations in the particle motion are necessary if any significant amount of energy is to be damped in the plasma. Equation (36)

implies that, at frequencies in the MHz range or higher, direct dissipation of the wave by collisions will be negligible. In order to magnify collisional absorption one has recourse to *resonances*. Under resonance conditions, a small excitation will create either a huge response in the particle's motion (*wave-particle resonance*) or large wave-field build-up (*wave resonance*). In Fig.1 the "absorption zone" is the region where such a resonance takes place (the shape is of course only symbolic).

III.B. Wave resonances..

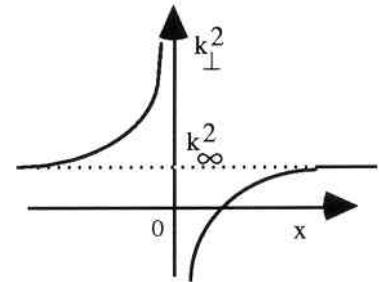
Wave resonances have been discussed in a previous lecture by Westerhof². Assuming that k_{\parallel} is for the essential determined by the antenna system, and thus fixed, and we shall characterise the resonance by $k_{\perp} \rightarrow \infty$. An example of wave resonance is that of the fast wave, Eq.(14) when

$$k_0^2 S - k_{\parallel}^2 = 0 \quad (37)$$

We shall examine in more detail how S varies across the plasma and gives rise to the ion-ion hybrid resonances in the ICRF subsequently¹⁶. Here, we are interested only in understanding how a wave resonance can give rise to absorption. It is sufficient to notice that, as we proceed from the plasma edge to the plasma inside, e.g. along the major radius direction (x), S will vary because both the magnetic field and the plasma density vary. As we shall see later in more detail, the simplest kind of resonance (assumed located at $x=0$) can be schematically represented by the wavevector variation

$$k_{\perp FW}^2 = k_{\infty}^2 - \frac{\alpha}{x} \equiv K^2(x) \quad (38)$$

with α a positive constant. This variation is schematically represented on Fig.5. Except in the vicinity of $x=0$, the wave is propagating with constant $k_{\perp} \approx \pm k_{\infty}$. If



the wave approaches the resonance from the left, it will slow-down more and more (its group velocity goes to zero) and come to rest at $x=0$. In this simple WKB picture, all wave fronts coming from the left pile-up at the resonance leading to a large increase in wave amplitude. In addition, as the wave field oscillates a large number of times before propagating any significant amount of distance toward the resonance, it is obvious

Fig. 5 The simplest example of a wave resonance.

that the effects of any damping mechanism will be considerably magnified in the vicinity of the resonance.

This simple picture has to be corrected for wave-specific effects. Indeed, part of the power will tunnel through the evanescent region (where $k_{\perp}^2 < 0$) and continue propagating on the other side ($x \rightarrow \infty$). The non-tunnelled fraction is absorbed at the resonance $x=0$. This can be seen from Eq.(14.1) with the wavevector from Eq.(38):

$$\frac{d^2 E_y}{dx^2} + K^2(x) E_y = 0 \quad (39)$$

By multiplying Eq.(39) by the complex conjugate of the electric field E_y^* and taking the imaginary part, we get a conservation law for the electromagnetic power flux:

$$\frac{d}{dx} \left[\text{Im}(E_y^* \frac{dE_y}{dx}) \right] + \text{Im}(K^2) |E_y|^2 = 0 \quad (40)$$

Indeed, from Eqs.(13), we see that the electromagnetic power flux in the x -direction, i.e. the x -component of the Poynting vector is:

$$\begin{aligned} P_x &= \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*)_x = \frac{1}{2} \text{Re}(E_y H_z^*) \\ &= \frac{1}{2} \text{Re}(E_y \frac{-1}{i\omega\mu_0} \frac{dE_y^*}{dx}) = \frac{-1}{2\omega\mu_0} \text{Im}(E_y \frac{dE_y^*}{dx}) \end{aligned} \quad (41)$$

which means that Eq.(40) is indeed a power conservation law that we can rewrite

$$\frac{dP_x}{dx} = - \frac{\text{Im}(K^2)}{2\omega\mu_0} |E_y|^2 \quad (42)$$

When K^2 is real, there is no loss anywhere and this equation implies that the power, is conserved. In the case of Eq.(38), integration across the singularity on the real x axis would be meaningless. One must remind here, that causality requires $\text{Im}(\omega)$ to have a small positive imaginary part² (ν) such that the pole in Eq.(38) is slightly off the x -axis, and the singularity can be represented according to the Plemelj formula

$$\frac{1}{x} = \mathcal{P}\left(\frac{1}{x}\right) \pm i\pi\delta(x) \quad (43)$$

where the symbol \mathcal{P} reminds that the principal part has to be taken when the integration is performed. The imaginary part represents half the residue at the pole and, in any practical case, the sign will follow from $\nu > 0$.

Accordingly, upon integration over x Eq.(41) implies that the power is constant for $x \neq 0$ and jumps abruptly at the crossing of the resonance. In the case of a cold or maxwellian plasma the jump correspond to a decrease in power, i.e. to wave energy absorption (by the plasma). This will be shown explicitly for the case of the ion-ion hybrid in a subsequent lecture¹⁶. Power is thus absorbed at the resonance notwithstanding the fact that no absorption mechanism was explicitly considered in the original equation (14). Introducing some small dissipation in the dielectric tensor, which for the essential is similar to

having a positive imaginary part $i\nu$ in ω , would lead to the same effect. For small values of ν , the absorption at the resonance is nearly independent of ν and keeps a non-zero value as $\nu \rightarrow 0$.

III.C. Wave-particle resonances – Landau damping.

Wave-particle resonances appear as resonant denominators in the integrand of the expression of the kinetic dielectric tensor². Physically, they result from the fact that, in their reference frame, particles see a constant electric field and are therefore uniformly accelerated. Such a singular phenomenon can appear only in an approximate treatment of the problem, namely in the linearised approximation. For example let us consider, as in Fig.6 a particle moving at the constant velocity v in an electric field directed along v and propagating at the phase velocity ω/k . The equations of motion of this particle

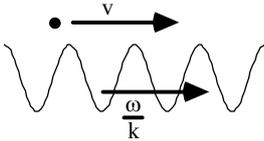
$$m \frac{dv}{dt} = ZeE \cos(kx - \omega t); \quad \frac{dx}{dt} = v$$


Fig. 6 Wave-particle resonance.

can be solved exactly and yield two types of solutions¹⁷: (i) passing particles having a negative velocity or a too large positive velocity travel at the

average speed v with an oscillation superposed on it, (ii) particles which travel at a velocity close to the phase velocity are trapped in the potential well of the wave and travel at the average velocity ω/k with a superposed oscillation which is the bouncing inside the well. These trajectories are integrable and there is no such thing as a “resonant motion” in the wave. (However a closer look reveals that there is a separatrix in phase space, corresponding to the trapped/passing boundary, which is extremely sensitive to perturbations, but this is another story). However, when we linearise the problem we arrive at the equation of motion:

$$m \frac{dv_1}{dt} = ZeE \cos(kx - \omega t) = ZeE \cos[(kv - \omega)t] \quad (45)$$

where we have used in the expression for the field the unperturbed particle motion $x=vt$. The perturbed velocity v_1 is oscillatory and there is no energy exchange between the particles and the wave as long as they have a different zero-order velocity $v \neq \omega/k$. But if they have the same velocity, the particle sees a constant electric field and is uniformly accelerated:

$$v_1 = \frac{ZeE}{m} t \quad (46.1)$$

An alternative way of looking at the same phenomenon is the Fourier transform approach, which here reduces to looking only at the periodic solutions of the problem:

$$v_1 = \frac{ZeE}{m} \frac{e^{i(kv-\omega)t}}{i(kv-\omega)} \quad (46.2)$$

It can be seen that the singularity in the spectrum appearing at the resonance corresponds to the secularity ($\propto t$) in the time-representation. The corresponding absorption is known as *Landau damping*.

III.D. Wave-particle resonances – Transit time magnetic pumping

Another frequently quoted non-collisional absorption mechanism is *transit time magnetic pumping* or TTMP. It is similar to Landau damping, except that it is due to a propagating magnetic –rather than electric- wave modulation:

$$B_1 = \delta B \cos(kx - \omega t) \quad (47)$$

superposed on the static field B_0 . This modulation gives rise to the $\mu \nabla B$ force acting on the particle's magnetic moment μ , according to the equation for the motion along the magnetic field lines:

$$m \frac{dv}{dt} = -\mu \nabla B \quad (48)$$

which is similar to Eq.(43).

III.E. Wave-particle resonances – cyclotron damping.

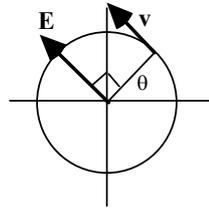


Fig.7. Cyclotron resonance.

Another type of wave-particle resonance is the cyclotron resonance (Fig.7). Assume that the particle is rotating at the cyclotron frequency $\theta = \omega_c t$ and that we apply a rotating electric field at the same frequency $\omega = \omega_c$ with a component along the particle velocity v : the particle's perpendicular energy will increase

linearly with time.

This very sketchy analysis puts into light a weakness of the linearised approach. Indeed what will happen in reality is not that uniform acceleration will take place indefinitely but rather that the accelerated particle will escape the resonant condition and thence terminate the resonant process. Resonant absorption can only continue if, once this particle has left the interaction area, it is replaced by a fresh one that can continue the resonant interaction. In terms of distribution functions, this means that the wave will produce velocity diffusion such as to empty the interaction region from particles, which in practice means a flattening of the distribution function in the interaction zone. Once the distribution is flat the interaction has stopped. Collisions or other processes like stochasticity counteract this tendency by restoring the gradients. If the latter processes are strong, the distribution function can remain maxwellian, however,

generally speaking, the distribution function of a particle population heated by a resonance process is not maxwellian.

IV. QUASILINEAR DIFFUSION AND TAILS

In the previous sections, we have seen that the RF power could be absorbed by the plasma via wave-particle or wave resonances. In both cases, the wave equation tells us that the wave will be damped while travelling in the plasma but leaves open the question: where is this power going to?

A wave resonance corresponds to infinity in physical variables. At resonance, not only the wave vector goes to infinity but field components as well, as is clear from the solution of the wave Eq.(14), which can be expressed analytically in terms of Bessel functions¹⁷. Such infinities are the sign that some smallness hypothesis is violated and that additional terms should have been retained in the wave equation. Retaining these terms changes the resonance into a mode conversion¹⁸ whereby the initial low- k_{\perp} wave is converted into high- k_{\perp} branch. Ultimately, the latter can only be absorbed through collisional or non-collisional (i.e. wave-particle resonance) processes.

The existence of absorption through wave-particle resonance manifests itself by the presence of an anti-hermitian part of the magnetised hot-plasma dielectric tensor². The classical expression of the dielectric tensor is obtained by assuming that the unperturbed distribution function is a maxwellian. We shall see that this is only an approximation and that the heating process only takes place with a sub-class of plasma particles and necessarily leads to some deformation of the distribution function of the heated population.

The corresponding theory is called *quasilinear* theory. The derivation of the quasilinear theory in the general case has been addressed in previous lectures^{1,3}. Here, we only want to outline in which way absorption and “heating” i.e. temperature increase, are inter-related. We do this for the simplest case, that of Landau damping in unmagnetised plasma.

We start with Vlasov's equation in one dimension

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial z} + \frac{ZeE}{m} \frac{\partial f}{\partial v} = 0 \quad (49)$$

where $E=E_1$ is the perturbed electric field (there is no equilibrium electric field). We decompose the distribution function into a slowly varying part (f_0), both in time and space, and a perturbed part (f_1), $f=f_0+f_1$, and insert this expression in Eq.(49). In order to isolate the slowly

varying part of the distribution function, we average this equation over time (many wave periods) and space (many wavelengths). Denoting by $\langle \cdot \rangle$ this averaging operation, we obtain:

$$\frac{\partial \mathcal{f}_0}{\partial t} + v \langle \frac{\partial \mathcal{f}_0}{\partial z} \rangle + \frac{Ze}{m} \langle E \frac{\partial \mathcal{f}_1}{\partial v} \rangle = 0 \quad (50)$$

where we have used the fact that $\langle E \rangle = \langle f_1 \rangle = 0$. The second term in this equation is zero for uniform plasma and we are left with an equation that determines the evolution of the equilibrium distribution function under the action of the first-order perturbations:

$$\frac{\partial \mathcal{f}_0}{\partial t} = -\frac{Ze}{m} \langle E \frac{\partial \mathcal{f}_1}{\partial v} \rangle \quad (51)$$

The second term in this equation is the *quasilinear* term. In order to write it down explicitly, we must solve the equation for the perturbation of the distribution function f_1 which we obtain by subtracting Eq.(51) from Eq.(49):

$$\frac{\partial f_1}{\partial t} + v \frac{\partial f_1}{\partial z} + \frac{ZeE}{m} \frac{\partial \mathcal{f}_0}{\partial v} = 0 \quad (52)$$

in which we recognise a simplified version of the linearised Vlasov equation². Fourier-transforming in space and Laplace-transforming in time ($f \rightarrow \exp[i(kz - \omega t)]$), we easily obtain the solution of this equation:

$$f_1 = -\frac{iZeE}{m(\omega - kv)} \frac{\partial \mathcal{f}_0}{\partial v} \quad (53)$$

Inserting this expression into Eq.(51), we obtain, noting that the average of two oscillating quantities $u(t)$ and $v(t)$ is $\langle u(t)v(t) \rangle = (1/2)\text{Re}(uv^*)$:

$$\frac{\partial \mathcal{f}_0}{\partial t} = -\frac{Ze}{m} \frac{1}{2} \text{Re} \left[E^* \frac{\partial}{\partial v} \frac{-iZeE}{m(\omega - kv)} \frac{\partial \mathcal{f}_0}{\partial v} \right] \quad (54)$$

$$= -\frac{Z^2 e^2 |E|^2}{2m^2} \frac{\partial}{\partial v} \text{Im} \left[\frac{1}{\omega - kv} \right] \frac{\partial \mathcal{f}_0}{\partial v}$$

This equation can be given a Fokker-Planck type form:

$$\frac{\partial \mathcal{f}_0}{\partial t} = \frac{\partial}{\partial v} \left(D \frac{\partial \mathcal{f}_0}{\partial v} \right) ; \quad D = -\frac{Z^2 e^2 |E|^2}{2m^2} \text{Im} \left[\frac{1}{\omega - kv} \right] \quad (55)$$

where D is the *quasilinear diffusion coefficient*. Note that by a formula similar to Eq.(43), the imaginary part appearing in the expression for D is nothing else than $\delta(\omega - kv)$. This implies that only the particles which are in resonance will be pushed in velocity space by the heating process.

This evolution equation for the distribution function allows to estimate the heating power going to the particles:

$$P_p = \frac{\partial}{\partial t} \int_{-\infty}^{\infty} \frac{1}{2} m v^2 f_0 dv = \int_{-\infty}^{\infty} \frac{1}{2} m v^2 \frac{\partial}{\partial v} \left(D \frac{\partial \mathcal{f}_0}{\partial v} \right) dv \quad (56)$$

$$= -m \int_{-\infty}^{\infty} v D \frac{\partial \mathcal{f}_0}{\partial v} dv$$

or, explicitly

$$P_p = \frac{Z^2 e^2 |E|^2}{2m} \int_{-\infty}^{\infty} \text{Im} \left[\frac{v}{\omega - kv} \right] \frac{\partial \mathcal{f}_0}{\partial v} dv \quad (47)$$

On the other hand, we can also compute the power dissipated by the waves, which follows from Maxwell's equations:

$$P_w = \frac{1}{2} \text{Re} [E^* j_1] \quad (58)$$

and we can compute the perturbed particle current from the perturbed distribution function Eq.(53):

$$j_1 = Ze \int_{-\infty}^{\infty} v f_1 dv = Ze \int_{-\infty}^{\infty} v \left[-\frac{iZeE}{m(\omega - kv)} \frac{\partial \mathcal{f}_0}{\partial v} \right] dv \quad (59)$$

$$= -\frac{iZ^2 e^2 E}{m} \int_{-\infty}^{\infty} \frac{v}{\omega - kv} \frac{\partial \mathcal{f}_0}{\partial v} dv$$

Accordingly,

$$P_w = \frac{Z^2 e^2 |E|^2}{2m} \int_{-\infty}^{\infty} \text{Re} \left[\frac{-iv}{\omega - kv} \right] \frac{\partial \mathcal{f}_0}{\partial v} dv \quad (60)$$

$$= \frac{Z^2 e^2 |E|^2}{2m} \int_{-\infty}^{\infty} \text{Im} \left[\frac{v}{\omega - kv} \right] \frac{\partial \mathcal{f}_0}{\partial v} dv$$

and we have the important identity

$$P_w = P_p \quad (61)$$

which expresses the fact that all the power that is lost by the waves is gained by the particle population. Although this might appear trivial in this present simple case, it is usually not granted in more complex theories where wave propagation and absorption on one side, and quasilinear diffusion on the other, are treated distinctly, using different approximations. The lack of consistency between the latter can break the equality Eq.(61)⁹.

V. THERMALISATION

If Eq.(55) is solved as it is, it can lead to a time-asymptotic stationary solution only if the energy transferred by the wave to the particles is zero. Indeed, its r.h.s. corresponds to power input to the plasma and there is no loss term to balance it. Therefore the stationary solution of Eq.(55) must exhibit a *quasilinear plateau*, i.e. a zones around the resonant velocity where f_0 is flat ($\partial \mathcal{f}_0 / \partial v = 0$). This ensures that the heating power vanishes, see Eq.(57).

In a situation where there is stationary power transfer to the plasma, the evolution equation for the particle's distribution functions, Eq.(55) is thus lacking a loss term. This is the collision term, which we denote by $C(f_0)$. It includes collisions on all particle species, including the heated ones. It also implies that the distribution function of the heated species will tend to relax to a maxwellian. From the different contributions to $C(f_0)$, one can then compute the power transfer to the different plasma components and the resulting temperature increases. The

complete equation for the evolution of f_0 should thus be written

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial v} \left(D \frac{\partial f_0}{\partial v} \right) + C(f_0) \quad (62)$$

This collisional thermalisation process is in close relation with the slowing down of fast NBI ions discussed earlier⁴

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